# EE105 – Fall 2014
# Microelectronic Devices and Circuits


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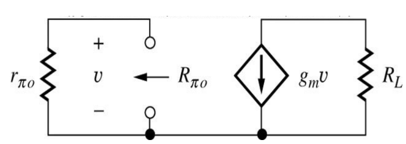
## Common-Emitter Amplifier – $\omega_H$
## Open-Circuit Time Constant (OCTC) Method

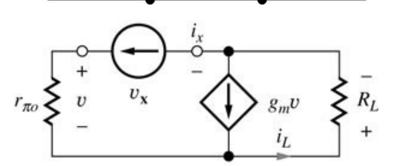
**At high frequencies, impedances of coupling and bypass capacitors are small enough to be considered short circuits. Open-circuit time constants associated with impedances of device capacitances are considered instead.**

$$\omega_H \cong \frac{1}{\sum_{i=1}^{m} R_{io} C_i}$$

**where $R_{io}$ is resistance at terminals of $i$th capacitor $C_i$ with all other capacitors open-circuited.**
**For a C-E amplifier, assuming $C_L = 0$**

$$R_{\pi 0} = r_{\pi 0}$$



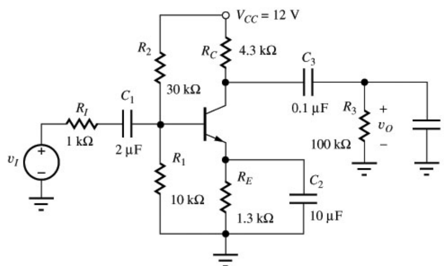



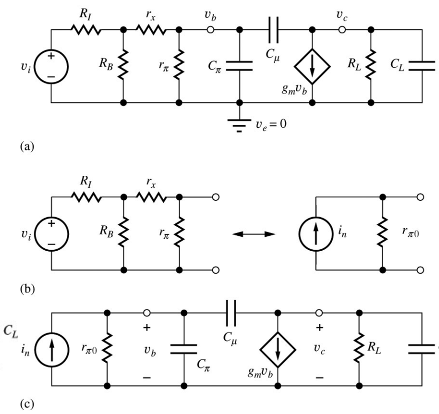

$$R_{\mu 0} = \frac{v_x}{i_x} = r_{\pi 0}\left(1 + g_m R_L + \frac{R_L}{r_{\pi 0}}\right)$$

$$\omega_H \cong \frac{1}{R_{\pi 0} C_\pi + R_{\mu 0} C_\mu} = \frac{1}{r_{\pi 0} C_T}$$

# Common-Emitter Amplifier High Frequency Response - Miller Effect

- **First, find the simplified small -signal model of the C-E amp.**
- **Replace coupling and bypass capacitors with short circuits**
- **Insert the high frequency small -signal model for the transistor**




$$r_{\pi 0} = r_\pi \| \left[ r_x + \left( R_B \| R_I \right) \right]$$



# Common-Emitter Amplifier – ωH High Frequency Response - Miller Effect (cont.)

**Input gain is found as**

$$A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_I + R_{in}} \cdot \frac{r_\pi}{r_x + r_\pi}$$

$$= \frac{R_1 \parallel R_2 \parallel (r_x + r_\pi)}{R_I + R_1 \parallel R_2 \parallel (r_x + r_\pi)} \cdot \frac{r_\pi}{r_x + r_\pi}$$

**Terminal gain is**

$$A_{bc} = \frac{v_c}{v_b} = -g_m \left( r_o \parallel R_C \parallel R_3 \right) \cong -g_m R_L$$

**Using the Miller effect, we find the equivalent capacitance at the base as:**

$$C_{eqB} = C_\mu (1 - A_{bc}) + C_\pi (1 - A_{be})$$

$$= C_\mu (1 - [-g_m R_L]) + C_\pi (1 - 0)$$

$$= C_\mu (1 + g_m R_L) + C_\pi$$

Chap 17-4

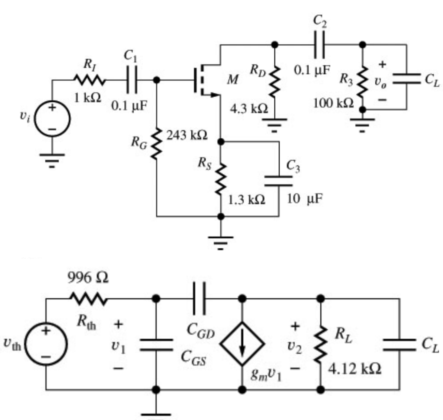
# Common-Emitter Amplifier – ωH
# High Frequency Response - Miller Effect (cont.)

- The total equivalent resistance at the base is $R_{eqB} = r_{\pi 0} = r_\pi \| \left[ r_x + \left( R_B \| R_I \right) \right]$
- The total capacitance and resistance at the collector are
$$C_{eqC} = C_\mu + C_L$$
$$R_{eqC} = r_o \| R_C \| R_3 = R_L$$
- Because of interaction through Cμ, the two RC time constants interact, giving rise to a dominant pole.
$$\omega_{p1} = \frac{1}{r_{\pi 0}\left[ C_\pi + C_\mu \left(1 + g_m R_L\right)\right] + R_L \left( C_\mu + C_L \right)}$$
$$\omega_{p1} = \frac{1}{r_{\pi 0} C_T} \quad \text{where}$$
$$C_T = C_\pi + C_\mu \left(1 + g_m R_L\right) + \frac{R_L}{r_{\pi 0}}\left( C_\mu + C_L \right)$$
$$r_{\pi 0} = r_\pi \| \left[ r_x + \left( R_B \| R_I \right) \right]$$



# Common-Source Amplifier – $\omega_H$
# Open-Circuit Time Constants

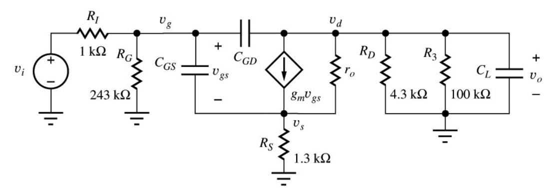
**Analysis similar to the C-E case yields the following equations:**

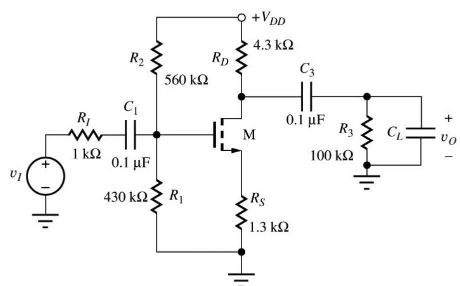



$$R_{th} = R_G \| R_I$$
$$R_L = R_D \| R_3 \| r_o$$
$$v_{th} = v_i \frac{R_G}{R_I + R_G}$$
$$C_T = C_{GS} + C_{GD}\left(1 + g_m R_L\right) + \frac{R_L}{R_{th}}\left( C_{GD} + C_L \right)$$
$$\omega_{P1} = \frac{1}{R_{th} C_T} \qquad \omega_{P2} = \frac{g_m}{C_{GS} + C_L}$$
$$\omega_Z = \frac{g_m}{C_{GD}}$$

# C-S Amplifier High Frequency Response Source Degeneration Resistance

First, find the simplified small -signal model of the C-S amp.




Recall that we can define an effective $g_m$ to account for the unbypassed source resistance.

$$g'_m = \frac{g_m}{1+g_m R_S}$$



# C-S Amplifier High Frequency Response Source Degeneration Resistance (cont.)

Input gain is found as

$$A_i = \frac{v_g}{v_i} = \frac{R_G}{R_i + R_G} = \frac{R_1 \| R_2}{R_i + R_1 \| R_2}$$

Terminal gain is

$$A_{gd} = \frac{v_d}{v_g} = -g_m'(R_{iD} \| R_D \| R_3) \cong \frac{-g_m\left(R_D \| R_3\right)}{1+g_m R_S}$$

Again, we use the Miller effect to find the equivalent capacitance at the gate as:

$$\begin{aligned} C_{eqG} &= C_{GD}(1-A_{gd}) + C_{GS}(1-A_{gs}) \\ &= C_{GD}\left[1 - \frac{\left(-g_m R_L\right)}{1+g_m R_S}\right] + C_{GS}\left(1 - \frac{g_m R_S}{1+g_m R_S}\right) \\ &= C_{GD}\left[1 + \frac{g_m\left(R_D \| R_3\right)}{1+g_m R_S}\right] + \frac{C_{GS}}{1+g_m R_S} \end{aligned}$$

# C-S Amplifier High Frequency Response Source Degeneration Resistance (cont.)

**The total equivalent resistance at the gate is**

$$R_{eqG} = R_G \| R_I = R_{th}$$

**The total capacitance and resistance at the drain are**

$$C_{eqD} = C_{GD} + C_L$$

$$R_{eqD} = R_{iD} \| R_D \| R_3 \cong R_D \| R_3 = R_L$$

**Because of interaction through $C_{GD}$, the two RC time constants interact, giving rise to the dominant pole:**

$$\omega_{p1} = \frac{1}{R_{th}\left[\dfrac{C_{GS}}{1+g_m R_S} + C_{GD}\left(1+\dfrac{g_m R_L}{1+g_m R_S}\right) + \dfrac{R_L}{R_{th}}(C_{GD}+C_L)\right]}$$

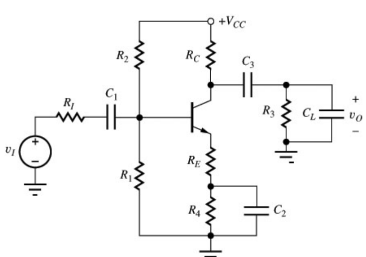
**And from previous analysis:**

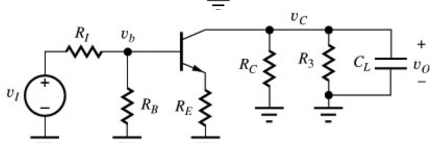
$$\omega_{p2} = \frac{g_m^{'}}{(C_{GS}+C_L)} = \frac{g_m}{(1+g_m R_S)(C_{GS}+C_L)} \qquad \omega_z = \frac{+g_m^{'}}{C_{GD}} = \frac{+g_m}{(1+g_m R_S)(C_{GD})}$$



# C-E Amplifier High Frequency Response Emitter Degeneration Resistance

**Analysis similar to the C-S case yields the following equations:**




$$R_{\pi 0} = R_{th} \| \left[r_\pi + (\beta_o + 1) R_E\right]$$

$$R_{th} = R_I \| R_B = R_I \| R_1 \| R_2 \qquad R_L = R_{iC} \| R_C \| R_3 \cong R_C \| R_3$$

$$\omega_{P1} = \frac{1}{R_{\pi 0} C_T}$$

$$\omega_{P1} = \frac{1}{R_{\pi 0}\left[\dfrac{C_\pi}{1+g_m R_E} + C_\mu\left(1+\dfrac{g_m R_L}{1+g_m R_E}\right) + \dfrac{R_L}{R_{\pi 0}}(C_\mu + C_L)\right]}$$

$$\omega_{P2} = \frac{g_m^{'}}{(C_\pi + C_L)} = \frac{g_m}{(1+g_m R_E)(C_\pi + C_L)}$$

$$\omega_z = \frac{+g_m^{'}}{C_\mu} = \frac{+g_m}{(1+g_m R_E)(C_\mu)}$$

# Gain-Bandwidth Trade-offs Using Source/Emitter Degeneration Resistors

Adding source resistance to the C-S (or C-E) amp causes gain to decrease and dominant pole frequency to increase.

$$A_{gd} = \frac{v_d}{v_g} = -\frac{g_m\left(R_D \parallel R_3\right)}{1+g_m R_S}$$

$$\omega_{p1} = \frac{1}{R_{th}\left[\frac{C_{GS}}{1+g_m R_S} + C_{GD}\left(1+\frac{g_m R_L}{1+g_m R_S}\right) + \frac{R_L}{R_{th}}\left(C_{GD}+C_L\right)\right]}$$

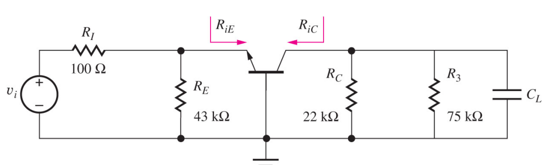
However, decreasing the gain also decreased the frequency of the second pole.

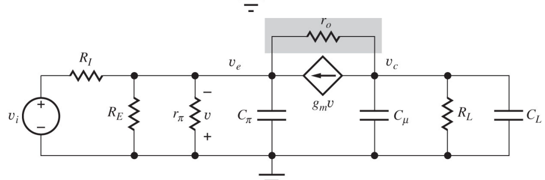
$$\omega_{p2} = \frac{g_m}{(1+g_m R_S)(C_{GS}+C_L)}$$

Increasing the gain of the C-E/C-S stage causes pole -splitting, or increase of the difference in frequency between the first and second poles.



# High Frequency Poles Common-Base Amplifier




$$A_i \cong \frac{1/g_m}{\left(1/g_m\right)+R_I} = \frac{1}{1+g_m R_I}$$

$$A_{ec} = g_m\left(R_{iC}\|R_L\right) \cong g_m R_L$$

$$R_{iC} = r_o\left[1+g_m\left(r_\pi\|R_E\|R_I\right)\right]$$

Since $C_\mu$ does not couple input and output, input and output poles can be found directly.

$$C_{eqE} = C_\pi$$

$$R_{eqE} = R_{iE}\|R_E\|R_I = \frac{1}{g_m}\|R_E\|R_I$$

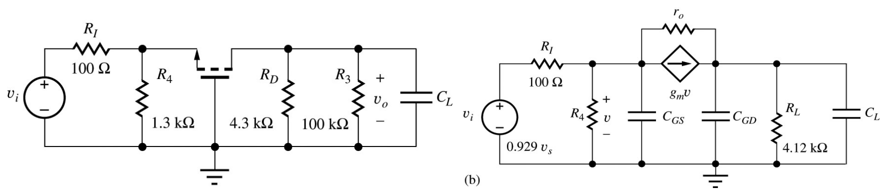
$$\omega_{P1} \cong \left[\left(\frac{1}{g_m}\|R_E\|R_I\right)C_\pi\right]^{-1} \cong \frac{g_m}{C_\pi} > \omega_T$$

$$C_{eqC} = C_\mu + C_L$$

$$R_{eqC} = R_{iC} \parallel R_L \cong R_L$$

$$\omega_{p2} = \frac{1}{(R_{iC} \parallel R_L)(C_\mu + C_L)} \cong \frac{1}{R_L(C_\mu + C_L)}$$

# High Frequency Poles
# Common-Gate Amplifier




**Similar to the C-B, since $C_{GD}$ does not couple the input and output, input and output poles can be found directly.**

$$C_{eqS} = C_{GS}$$

$$R_{eqS} = \frac{1}{g_m} \| R_4 \| R_I$$

$$\omega_{p1} = \frac{1}{\left(\frac{1}{g_m} \| R_4 \| R_I\right) C_{GS}} \cong \frac{g_m}{C_{GS}}$$

$$C_{eqD} = C_{GD} + C_L$$

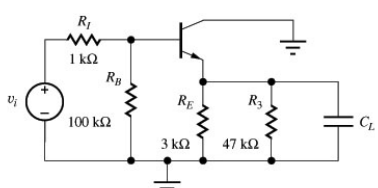
$$R_{iD} = r_o\left[1 + g_m\left(R_4 \| R_I\right)\right]$$

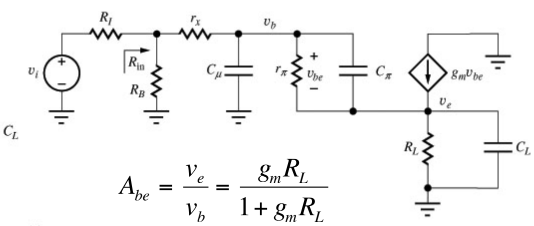
$$R_{eqD} = R_{iD} \| R_L \cong R_L$$

$$\omega_{p2} = \frac{1}{(R_{iD} \| R_L)(C_{GD} + C_L)} \cong \frac{1}{R_L(C_{GD} + C_L)}$$



# High Frequency Poles
# Common-Collector Amplifier




$$A_{be} = \frac{v_e}{v_b} = \frac{g_m R_L}{1 + g_m R_L}$$

$$C_{eqB} = C_\mu(1 - A_{bc}) + C_\pi(1 - A_{be}) = C_\mu(1 - 0) + C_\pi\left(1 - \frac{g_m R_L}{1 + g_m R_L}\right)$$

$$A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_i + R_{in}}$$

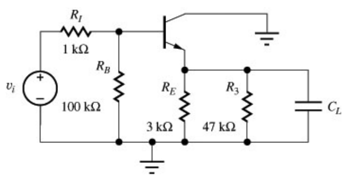
$$C_{eqB} = C_\mu + \frac{C_\pi}{1 + g_m R_L}$$

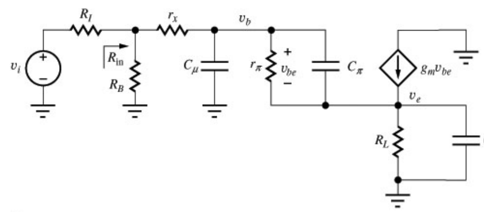
$$C_{eqE} = C_\pi + C_L$$

$$R_{eqB} = (R_{th} + r_x) \| R_{iB} = [(R_I \| R_B) + r_x] \| [r_\pi + (\beta_o + 1)R_L] = (R_{th} + r_x) \| [r_\pi + (\beta + 1)R_L]$$

$$R_{eqE} = R_{iE} \| R_L \cong \left(\frac{1}{g_m} + \frac{(R_{th} + r_x)}{\beta_o + 1}\right) \| R_L$$

# High Frequency Poles
# Common-Collector Amplifier (cont.)




**The low impedance at the output makes the input and output time constants relatively well decoupled, leading to two poles.**

$$\omega_{p1} = \frac{1}{\left(R_{th} + r_x\right) \| \left[r_\pi + (\beta_o + 1)R_L\right]\left(C_\mu + \dfrac{C_\pi}{1 + g_m R_L}\right)}$$

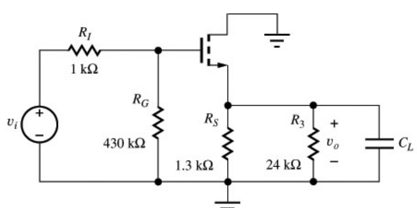
$$\omega_{p2} \cong \frac{1}{\left[\left(\dfrac{1}{g_m} + \dfrac{R_{th} + r_x}{\beta + 1}\right) \| R_L\right]\left(C_\pi + C_L\right)}$$

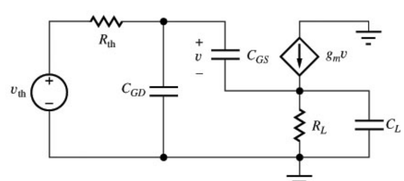
**The feed-forward high-frequency path through $C_\pi$ leads to a zero in the C-C response. Both the zero and the second pole are quite high frequency and are often neglected, although their effect can be significant with large load capacitances.**

$$\omega_z \cong \frac{g_m}{C_\pi}$$



# High Frequency Poles
# Common-Drain Amplifier



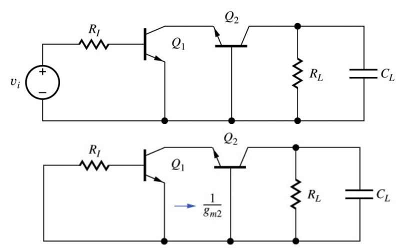

**Similar the the C-C amplifier, the C-D high frequency response is dominated by the first pole due to the low impedance at the output of the C-C amplifier.**

$$\omega_{p1} = \frac{1}{R_{th}(C_{GD} + \dfrac{C_{GS}}{1 + g_m R_L})}$$

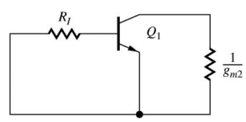
$$\omega_z \cong \frac{g_m}{C_{GS}}$$

$$\omega_{p2} = \frac{1}{\left(R_{iS} \| R_L\right)\left(C_{GS} + C_L\right)} \cong \frac{1}{\left(\dfrac{1}{g_m} \| R_L\right)\left(C_{GS} + C_L\right)}$$

# Frequency Response Cascode Amplifier




There are two important poles: the input pole for the C-E and the output pole for the C-B stage. The intermediate node pole can usually be neglected because of the low impedance at the input of the C-B stage. $R_{L1}$ is small, so the second term in the first pole can be neglected. Also note the $R_{L1}$ is equal to $1/g_{m2}$.




$$\omega_{pB1} = \frac{1}{r_{\pi 0} C_T} = \frac{1}{r_{\pi 01}\left(\left[C_{\mu 1}\left(1+\frac{g_{m1}R_{L1}}{1+g_{m1}R_{E1}}\right)+C_{\pi 1}\right]+\frac{R_{L1}}{r_{\pi 0}}\left[C_{\mu 1}+C_{L1}\right]\right)}$$

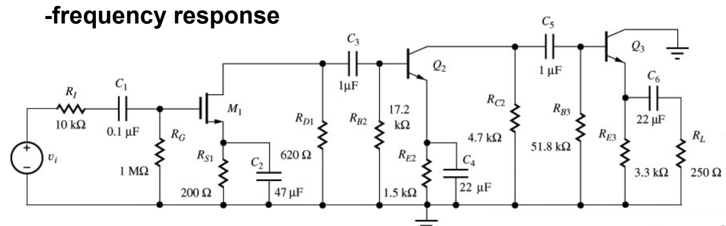
$$\omega_{pB1} = \frac{1}{r_{\pi 01}\left(\left[C_{\mu 1}\left(1+\frac{g_{m1}}{g_{m2}}\right)+C_{\pi 1}\right]+\frac{1/g_{m2}}{r_{\pi 01}}\left[C_{\mu 1}+C_{\pi 2}\right]\right)} \cong \frac{1}{r_{\pi 01}(2C_{\mu 1}+C_{\pi 1})}$$

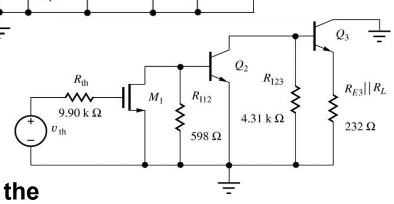
$$\omega_{pC2} \cong \frac{1}{R_L(C_{\mu 2}+C_L)}$$



# Frequency Response of Multistage Amplifier

- Problem: Use open-circuit and short-circuit time constant methods to estimate upper and lower cutoff frequencies and bandwidth.
- Approach: Coupling and bypass capacitors determine the low -frequency response; device capacitances affect the high -frequency response




- At high frequencies, ac model for the multi-stage amplifier is as shown.

# Frequency Response
# Multistage Amplifier Parameters (example)

**Parameters and operation point information for the example multistage amplifier.**

**TABLE 17.3**
Transistor Parameters

| | $g_m$ | $r_\pi$ | $r_o$ | $\beta_o$ | $C_{GS}/C_\pi$ | $C_{GD}/C_\mu$ | $r_x$ |
|---|---|---|---|---|---|---|---|
| $M_1$ | 10 mS | $\infty$ | 12.2 kΩ | $\infty$ | 5 pF | 1 pF | 0 Ω |
| $Q_2$ | 67.8 mS | 2.39 kΩ | 54.2 kΩ | 150 | 39 pF | 1 pF | 250 Ω |
| $Q_3$ | 79.6 mS | 1.00 kΩ | 34.4 kΩ | 80 | 50 pF | 1 pF | 250 Ω |



# Frequency Response
# Multistage Amplifier: High-Frequency Poles

**High-frequency pole at the gate of $M_1$: Using our equation for the C-S input pole:**

$$f_{P1} = \frac{1}{2\pi}\frac{1}{R_{th1}\left[C_{GD1}\left(1+g_m R_{L1}\right)+C_{GS1}+\frac{R_{L1}}{R_{th1}}\left(C_{GD1}+C_{L1}\right)\right]}$$

$$R_{L1} = R_{I12}\left\|\left(r_{x2}+r_{\pi 2}\right)\right\|r_{o1} = 598\Omega\left\|\left(250\Omega+2.39k\Omega\right)\right\|12.2k\Omega = 469\ \Omega$$

$$C_{L1} = C_{\pi 2}+C_{\mu 2}\left(1+g_{m2}R_{L2}\right)$$

$$R_{L2} = R_{I23}\left\|R_{in3}\right\|r_{o2} = R_{I23}\left\|\left(r_{x3}+r_{\pi 3}+\left(\beta_{o3}+1\right)\left(R_{E3}\|R_L\right)\right)\right\|r_{o2} = 3.33\ k\Omega$$

$$C_{L1} = C_{\pi 2}+C_{\mu 2}\left(1+g_{m2}R_{L2}\right) = 39pF+1pF\left[1+67.8mS\left(3.33k\Omega\right)\right] = 266\ pF$$

$$f_{P1} = \frac{1}{2\pi}\frac{1}{9.9k\Omega\left[1pF\left[1+0.01S\left(3.33k\Omega\right)\right]+5pF+\frac{469\Omega}{9.9k\Omega}\left(1pF+266pF\right)\right]} = 689\ kHz$$

## Frequency Response
## Multistage Amplifier: High-Freq. Poles (cont.)

**High-frequency pole at the base of $Q_2$: From the detailed analysis of the C-S amp, we find the following expression for the pole at the output of the $M_1$ C-S stage:**

$$f_{p2} = \frac{1}{2\pi}\frac{C_{GS1}g_{L1} + C_{GD1}(g_{m1} + g_{th1} + g_{L1}) + C_{L1}g_{th1}}{[C_{GS1}(C_{GD1} + C_{L1}) + C_{GD1}C_{L1}]}$$

**For this particular case, $C_{L1}$ ($Q_2$ input capacitance) is much larger than the other capacitances, so $f_{p2}$ simplifies to:**

$$f_{p2} \cong \frac{1}{2\pi}\frac{C_{L1}g_{th1}}{[C_{GS1}C_{L1} + C_{GD1}C_{L1}]} \cong \frac{1}{2\pi}\frac{1}{R_{th1}(C_{GS1} + C_{GD1})}$$

$$f_{p2} = \frac{1}{2\pi\left(9.9k\Omega\right)(5pF + 1pF)} = 2.68\ MHz$$



## Frequency Response
## Multistage Amplifier: High-Freq. Poles (cont.)

**High-frequency pole at the base of $Q_3$: Again, due to the pole-splitting behavior of the C-E second stage, we expect that the pole at the base of $Q_3$ will be set by equation 17.96:**

$$f_{p3} \cong \frac{g_{m2}}{2\pi[C_{\pi 2}(1 + \frac{C_{L2}}{C_{\mu 2}}) + C_{L2}]}$$

**The load capacitance of $Q_2$ is the input capacitance of the C-C stage.**

$$C_{L2} = C_{\mu 3} + \frac{C_{\pi 3}}{1 + g_{m3}\left(R_{E3} \parallel R_L\right)} = 1pF + \frac{50\,pF}{1 + 79.6mS(3.3k\Omega \parallel 250\Omega)} = 3.55\ pF$$

$$f_{p3} \cong \frac{67.8mS[1k\Omega/(1k\Omega + 250\Omega)]}{2\pi[39\,pF(1 + \frac{3.55pF}{1pF}) + 3.55pF]} = 47.7\ MHz$$

## Frequency Response
## Multistage Amplifier: $f_H$ Estimate

**There is an additional pole at the output of Q3, but it is expected to be at a very high frequency due to the low output impedance of the C-C stage. We can estimate $f_H$ from eq. 16.23 using the calculated pole frequencies.**

$$f_H = \frac{1}{\sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2} + \frac{1}{f_{p3}^2}}} = 667\ \text{kHz}$$

**The SPICE simulation of the circuit on the next slide shows an $f_H$ of 667 kHz and an $f_L$ of 530 Hz. The phase and gain characteristics of our calculated high frequency response are quite close to that of the SPICE simulation. It was quite important to take into account the pole-splitting behavior of the C-S and C-E stages. Not doing so would have resulted in a calculated $f_H$ of less than 550 kHz.**



## Frequency Response
## Multistage Amplifier: SPICE Simulation